Statistical Hadronization phenomenology...

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Based on: nucl-th/0510024,0509077,0509067,0503026 (Review coming shortly) In collaboration with
S. Jeon, J. Rafelski, J. Letessier

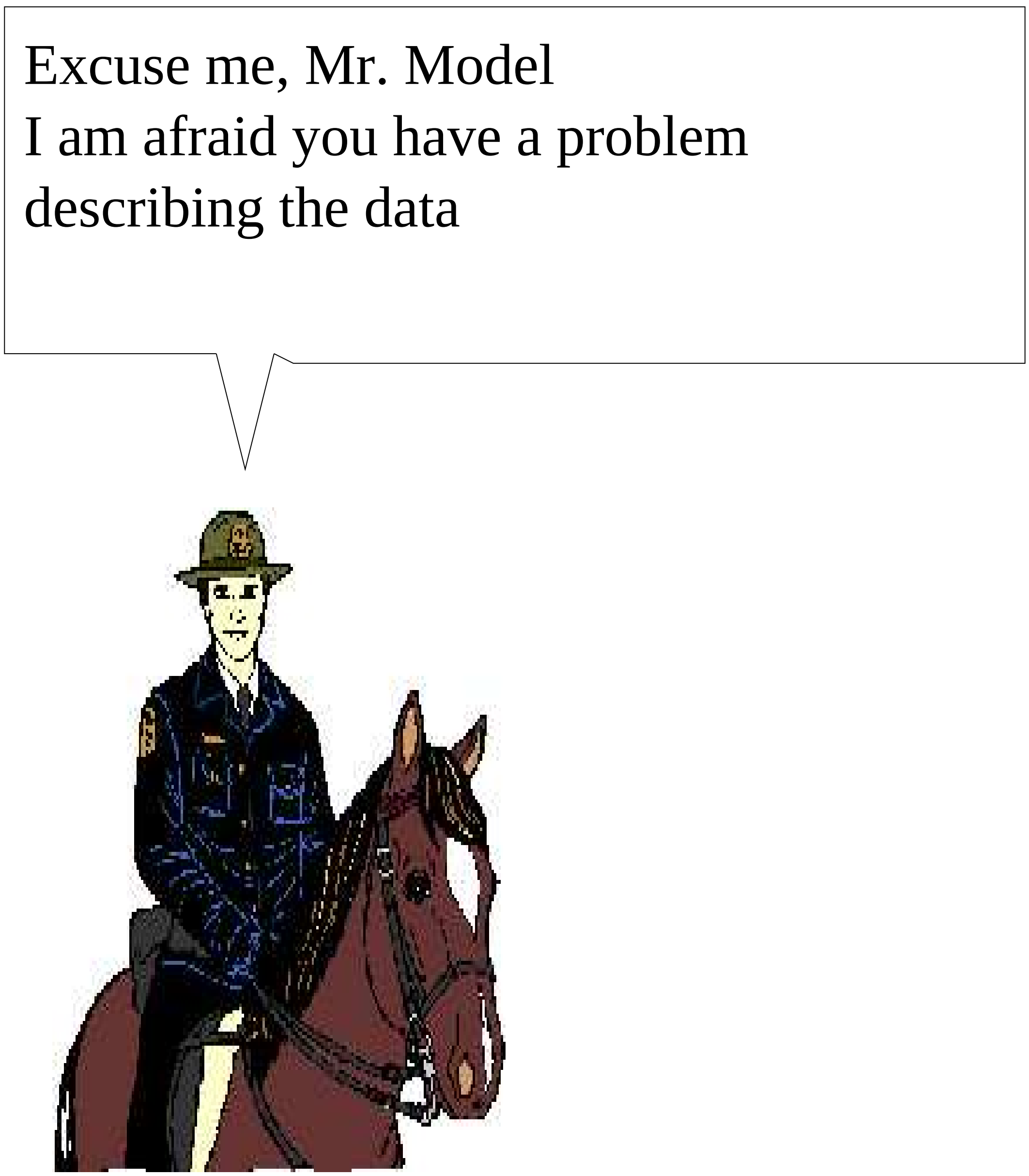
- The usefulness of fluctuations: They can provide an experimental answer to each of the questions below:
  - Is statistical hadronization really there?
  - What is the strangeness enhancement mechanism?
  - How significant are post freeze-out reinteractions?
  - Is there quark chemical non-equilibrium?
  - What is the chemical freeze-out temperature?

- The pitfalls of using fluctuations, and how to deal with them
  - Volume fluctuations
  - Global conservation laws
  - Detector acceptance corrections for primary particles
  - Detector acceptance corrections for resonances

- Conclusions and use SHARE!

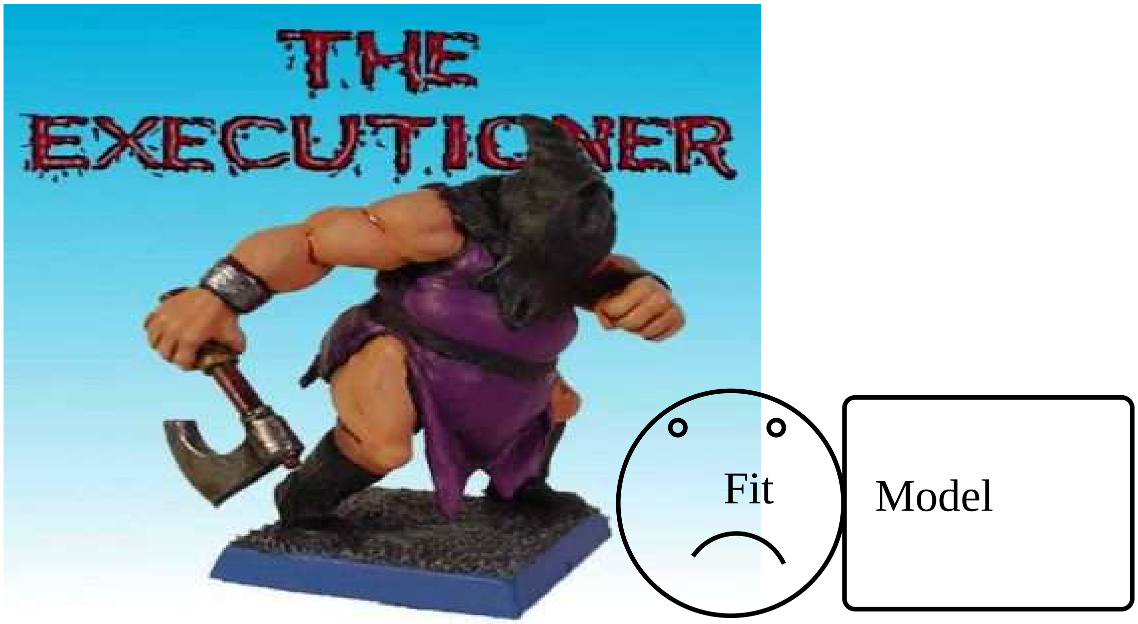
Part I
The usefulness of quantitative fluctuations studies

Canadian version/Version Canadienne



The usefulness of quantitative fluctuations studies

US version



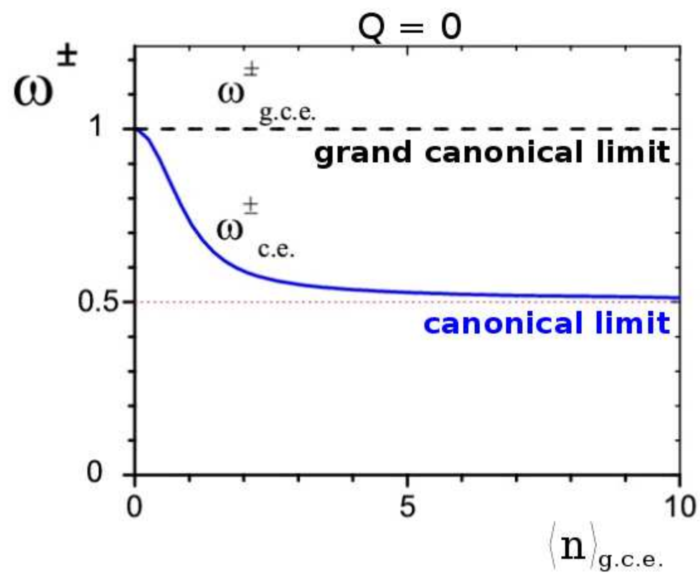
First question: Can we test statistical hadronization?
Fluctuations: Statistical mechanics falsifier

Statistical mechanics (in fact, all statistics)predicts a relationship between yields and fluctuations.
The validity of statistical mechanics is founded on fluctuations going to 0 in certain limits.
Measuring both yields and fluctuations → Falsifies all statistical models!

If, in a volume element small enough for the Grand Canonical ensemble to be appropriate, the same set of statistical parameters can not describe both yields and fluctuations, statistical model is wrong
i.e. Particle production not described by enthropy maximization.

Second question: What ensemble most appropriate?
Fluctuations: The ensemble-O-meter

The dependance of fluctuations on yields is Ensemble-specific (Begun,Gorenstein,Gazdzicki,Zozulya)



It is very unlikely for the incorrect ensemble to describe both yields and fluctuations with the same parameters

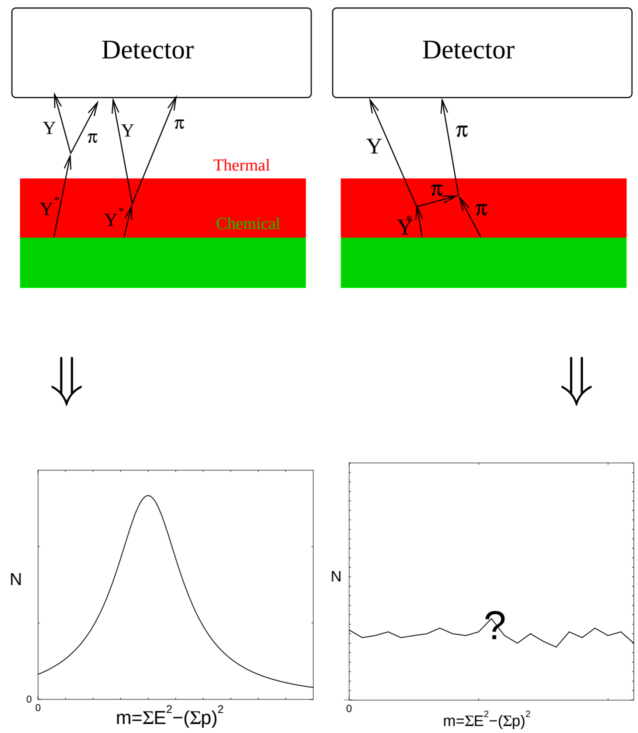
If canonical ensemble is a good description of strangeness in p-p collisions, than it has to describe strangeness fluctuations in p-p collisions with same T,V as yields

Third question: How much re-interaction between chemical and thermal freeze-out?

First answer: Resonances

$\frac{\Sigma^*}{\Lambda}, \frac{K^*}{K}, \frac{\Lambda(1520)}{\Lambda}, \frac{\Xi(1530)}{\Xi}, ...$Sensitive T probe

also susceptible to in-medium re-interactions



In general, rescattering will depend on Γ (dimensional analysis+optical theorem)

$$N_i\left(\frac{m_i}{T},\lambda\right) \rightarrow F\left[N_i\left(\frac{m_i}{T_{chem}},\lambda_{chem}\right),\Gamma_i\tau^{resc}\right]$$

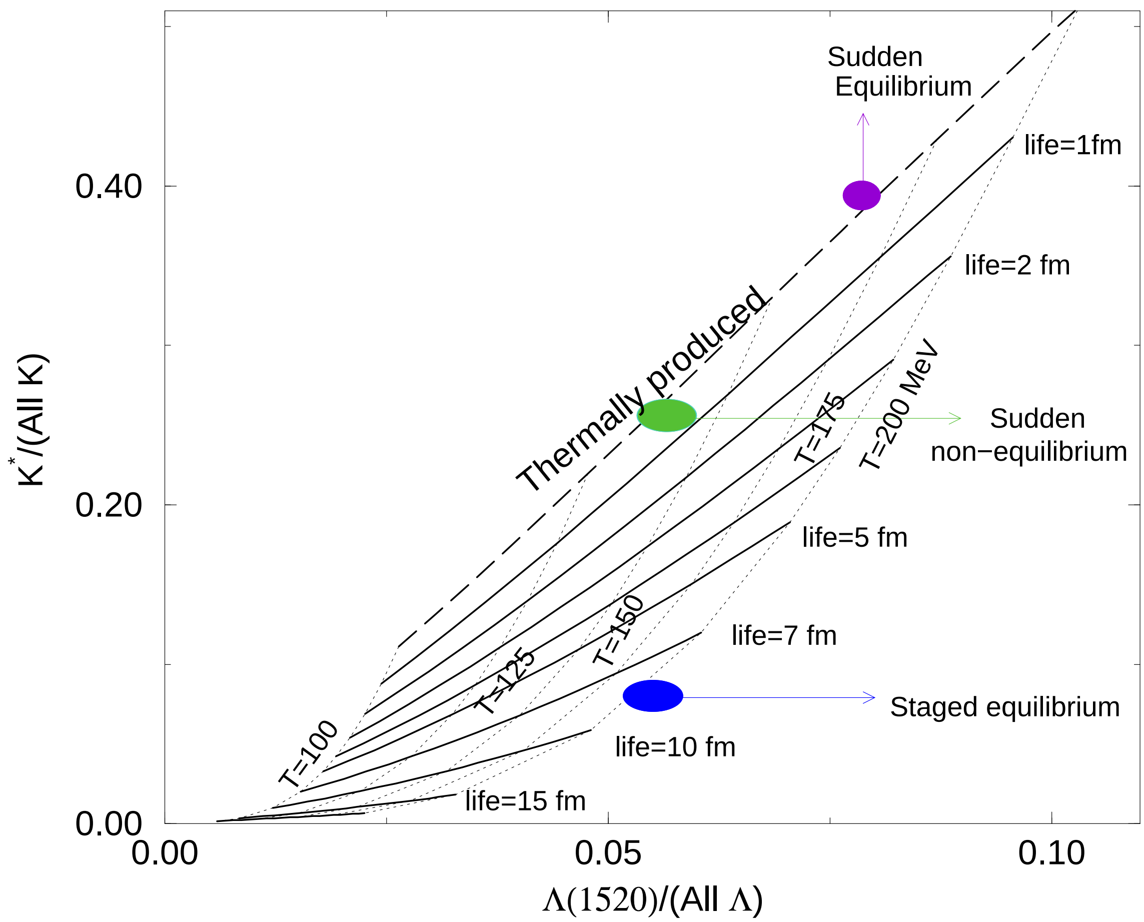
2 ratios, such as $\frac{\Sigma^*(1385)}{\Lambda}$ vs $\frac{K^*}{K} \Leftrightarrow T_{chem}$ and τ_{resc}

Rescattering model, GT and Rafelski, PLB, 509 239

$$\frac{dN^*}{dt} = -\Gamma N^*$$

$$\frac{d(N\pi)}{dt} = \Gamma N^* + (N\pi) < \sigma\gamma v > \frac{N_0}{V_0}\left(\frac{R_0}{R_0 + vt}\right)^3$$

- Observable $(N\pi)$ pairs created through decay and destroyed through rescattering
- Density $\frac{N_0}{V_0}$ fixed by statistical hadronization, R_0 by particle multiplicity, flow from spectral fits



- People doubt this since we neglected regeneration

- Semi classical approaches such as uRQMD drastically over-estimate n. of regenerated detectable particles by mass-shell assumption

But these are just words (and models!). We still have an ambiguity. Is there a experimental way to rule out either a fast freeze-out or a long reinteracting phase? Yes! Fluctuations

Yields and fluctuations: Reinteraction (or not)
Consider $Y^* \rightarrow Y\pi$

$\sigma_{Y/\pi}$ probes correlation of Y and π from Y^* at chemical freeze-out.

$$\sigma_{Y/\pi} = \frac{\left\langle (\Delta Y)^2 \right\rangle}{\left\langle Y \right\rangle^2} + \frac{\left\langle (\Delta \pi)^2 \right\rangle}{\left\langle \pi \right\rangle^2} - \frac{2}{\left\langle Y \right\rangle \left\langle \pi \right\rangle} \underbrace{\left\langle \Delta Y \Delta \pi \right\rangle}_{Y^* \rightarrow Y\pi}$$

(further rescattering/regeneration does not change the correlation.

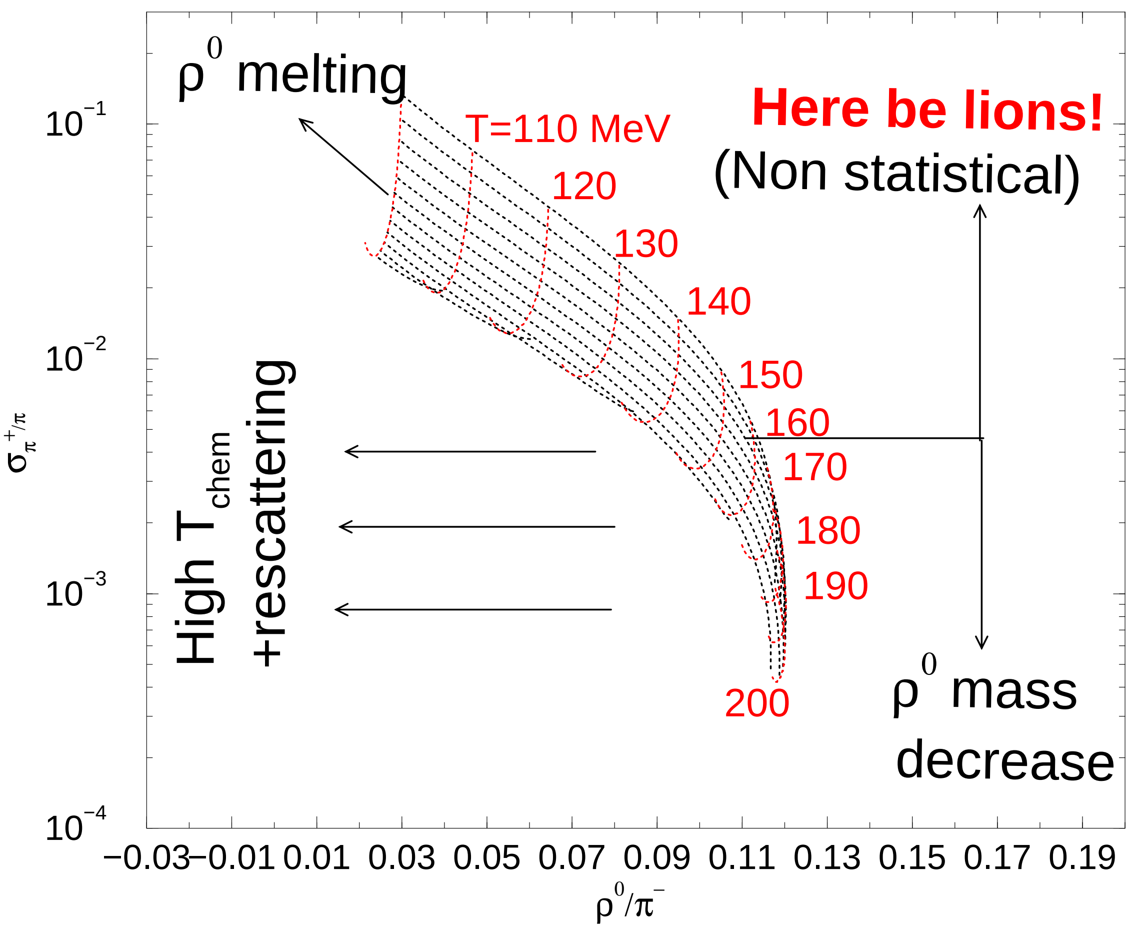
Y^*/Y **yield** probes Y^*at thermal freeze-out (after all rescattering.

So...

- If can fit stable particles and resonances and fluctuations in same fit $\rightarrow$ no reinteraction
- If Stable particles+ Fluctuations fit gives wrong value for resonances $\rightarrow$ magnitude of reinteraction

σ_{π^+/π^-} vs ρ^0/π^-

Probes (lack of?) reinteraction and mass modification separately

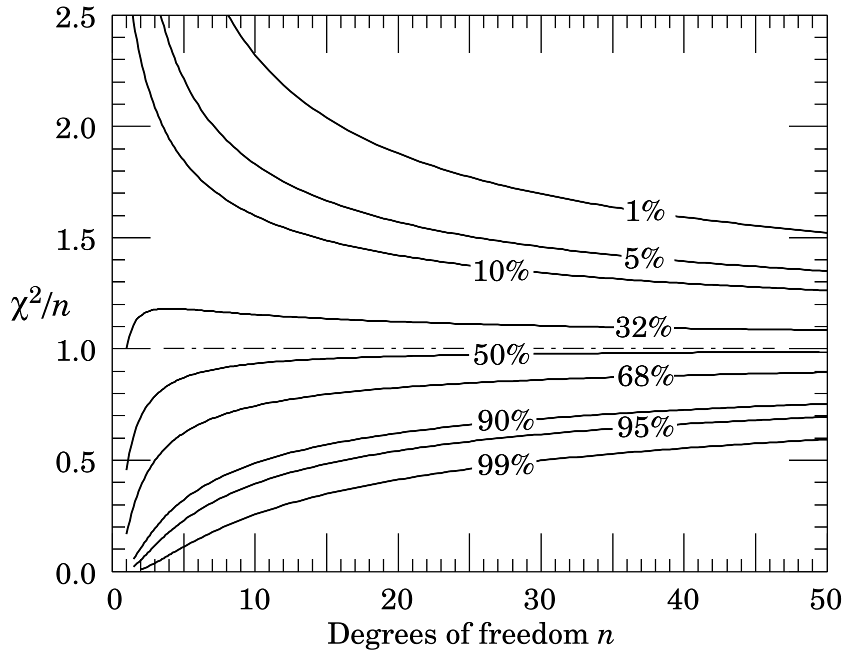


(I am cheating a bit here since σ_{π^+/π^-} contains a volume dependance... but as we will see, this is easy to get around!

Third and fourth questions

We heard about 2 statistical models!

Equilibrium statistical model	Non-equilibrium
oven-like	Explosion-like
High T (~ 165 MeV)	Supercooled (~ 140MeV)
Equilibrium ($\gamma_{q,s} = 1$)	Over-saturation ($\gamma_{q,s} > 1$)
Staged freeze-out	Sudden freeze-out
Resonances don't freeze-out at same T	Resonances freeze-out at same T
Strangeness systematics due to approach to thermodynamic limit (Canonical $\rightarrow$ GC)	Strangeness systematics due to phase transition γ_s/γ_q grows since more s/Q in QGP
No info on phase transition	First order or sharp cross-over
No info on early phase	Early phase probed



- Statistical significance, the probability of getting χ^2 with n DoF given that "your model is true", is a quantitative measure of your fit's goodness

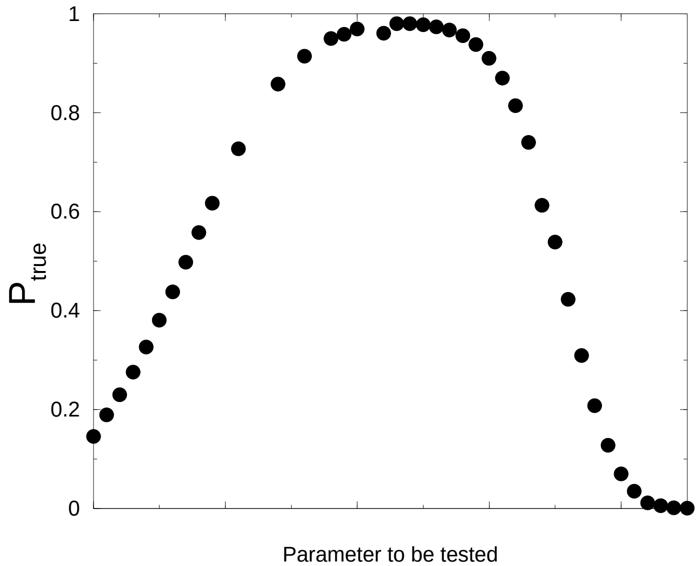
- models with different N_{dof} can be compared

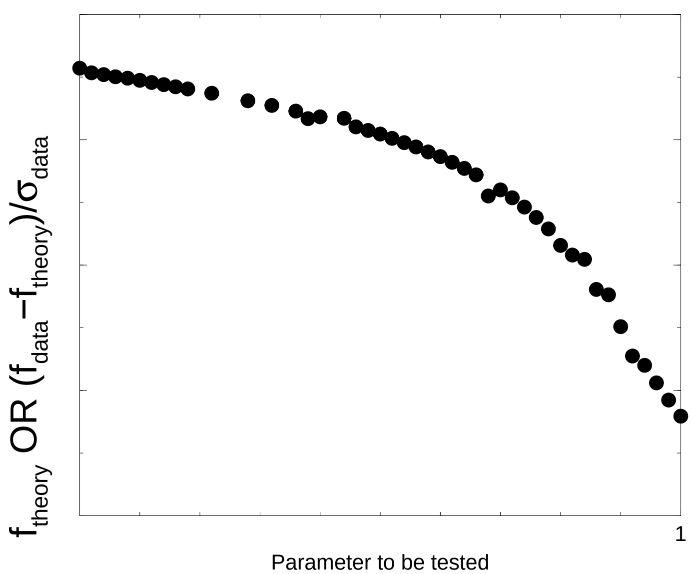
- With few DoF, "nice" looking graphs can have a very small statistical significance.

- It is said that you can fit an elephant with enough parameters. Maybe so, but if you are honest, you won't get a good statistical significance.

Non-trivial correlations/data-point sensitivity can be analyzed by Profiles in statistical significance

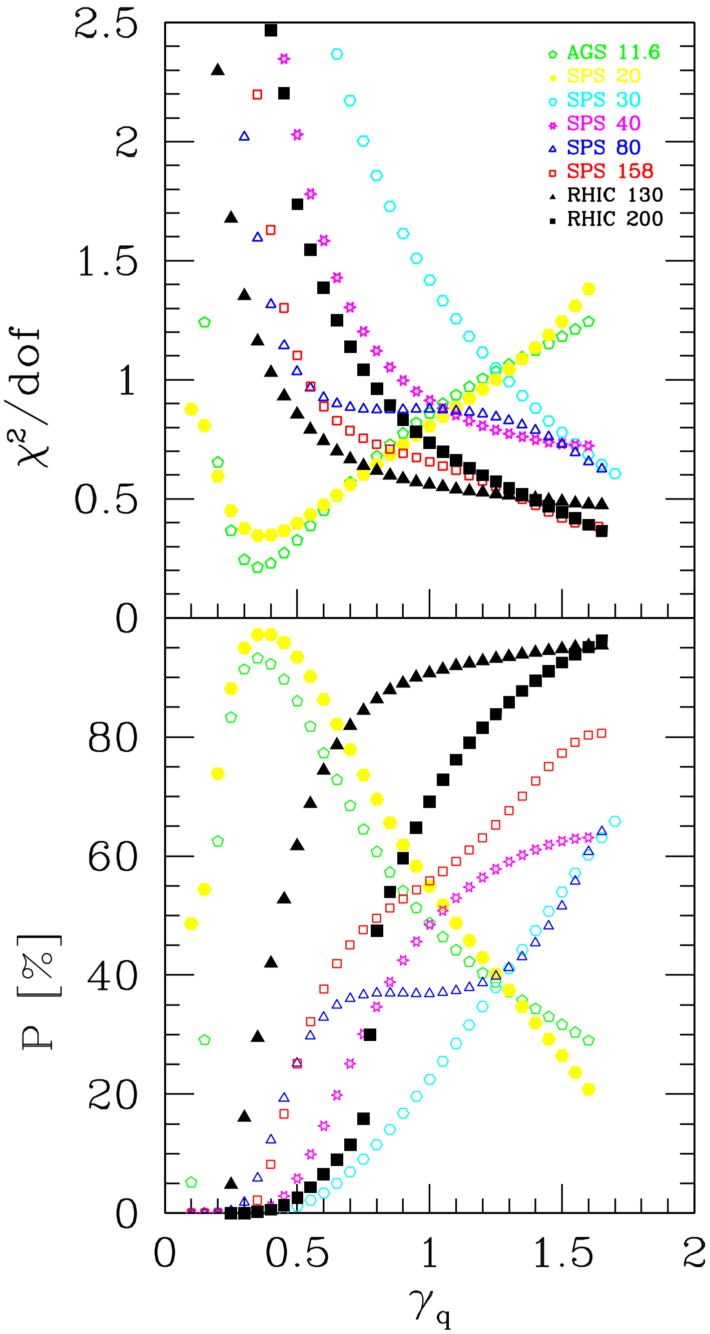
All other parameters at their best fit value for point in abscissa





Let's apply this to γ_q!

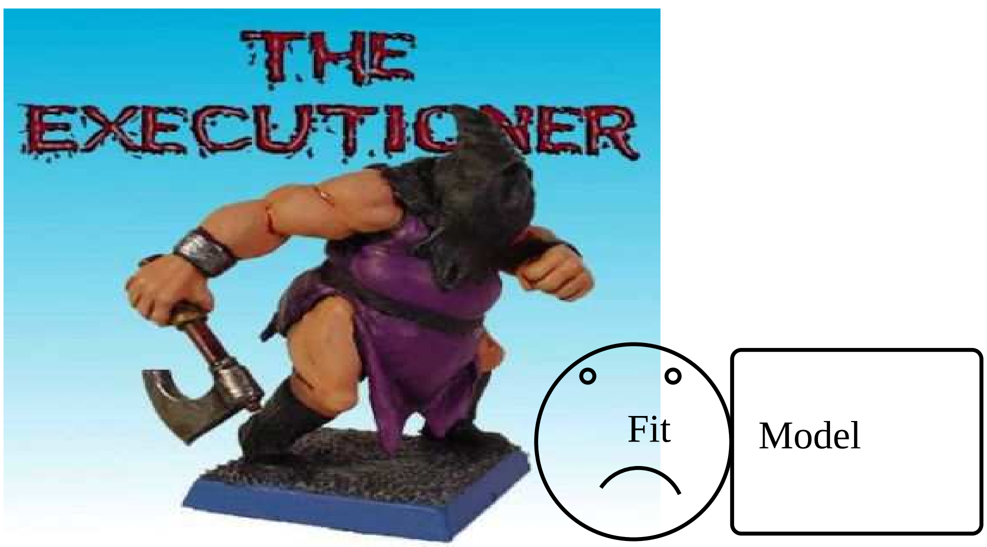
(Letessier and Rafelski, nucl-th/0504028)



- Maximum for SPS and RHIC is at $\gamma_q > 1$, suggesting this is probably not over-fitting
  - $\left(\frac{\gamma_s}{\gamma_q}\right)_{\gamma_q>1} > \left(\frac{\gamma_s}{\gamma_q}\right)_{\gamma_q=1} \Rightarrow$ More $\frac{\Lambda}{p}, \frac{\Xi}{\Lambda}, \frac{\Omega}{\Xi}$
  - Lower T$\Rightarrow$ less resonances agrees with Experiment

- But equilibrium not ruled out!.
  T and γ_q strongly correlated, making their individual determination difficoult

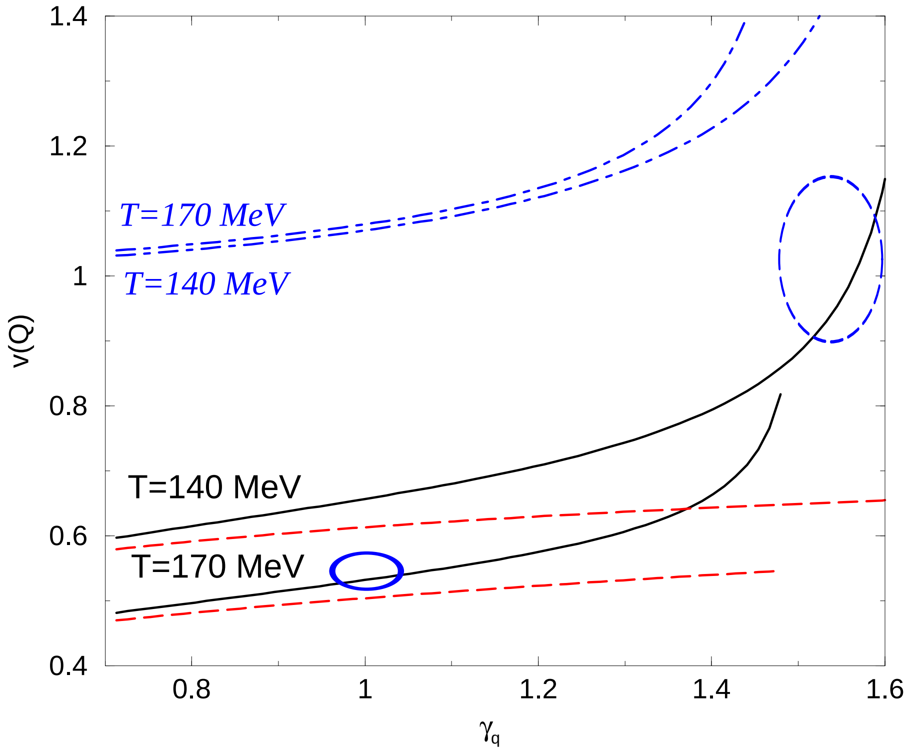
We need this guy:



ie, further data...

- That one EXPECTS statistical models to describe
- That is capable of determining γ_q,T, post-emission reinteraction.

Yields and Fluctuations: Non-equilibrium

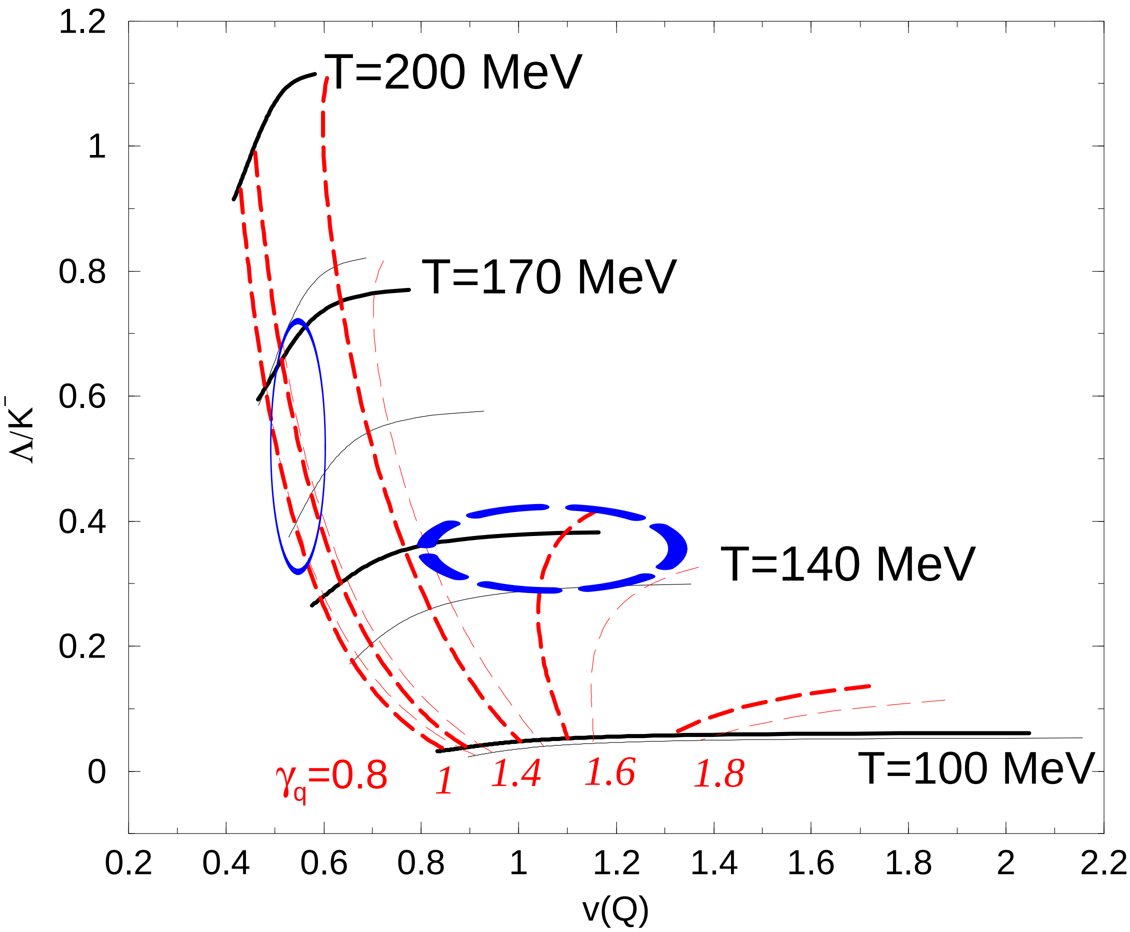


T increase $\Rightarrow$ π Fluctuations decrease because of enhanced resonance production
Resonances affect correlations

over-saturation ($\gamma_q > 1$) $\Rightarrow \pi$ Fluctuations increase faster than yields because of BE corrections

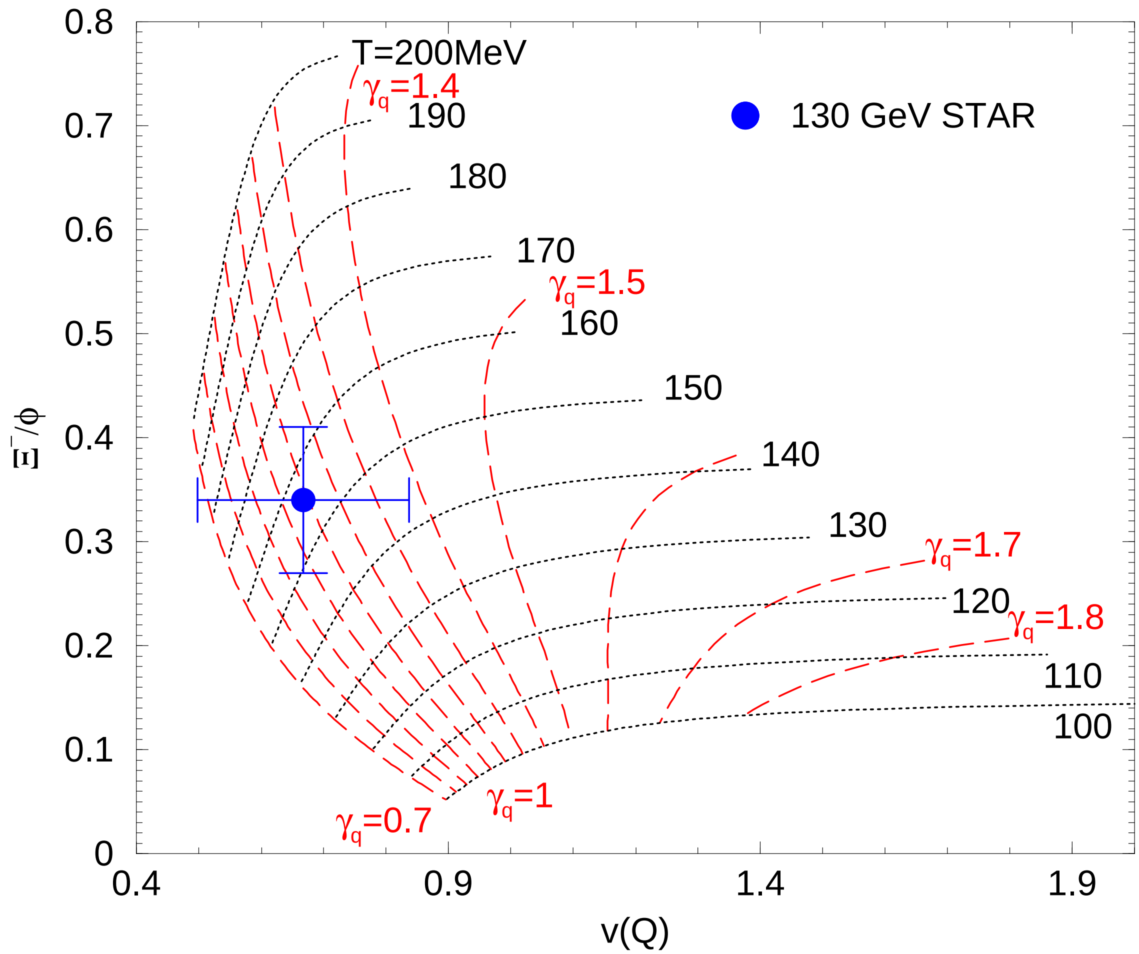
$$\gamma_q^2 e^{m_\pi/T} = 1-\epsilon \Rightarrow \frac{\langle N_\pi \rangle}{V} \sim \epsilon \qquad \frac{\langle (\Delta N_\pi)^2 \rangle}{V} \sim \epsilon^2$$

$\gamma_q > 1$ affects primordial fluctuations, so can not compensate for T v(Q)vs Λ/K^-



So T and γ_q decouple when both a yield a fluctuation are measured, One can not compensate for the other!

$v(Q)$ vs Ξ^-/ϕ



0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.4
0.9
1.4
1.9
v(Q)
Ξ⁻/φ
130 GeV STAR
T=200MeV
190
180
170
160
150
140
130
120
110
100
γq=1.4
γq=1.5
γq=1.7
γq=1.8
γq=1
γq=0.7

Part II

Why quantitative studies of fluctuations can be dangerous

Fluctuations are a lot more prone to systematic distortions than yields. If we are going to use them to kill models based on experimental data, we have to be extra careful!

A small problem: Volume fluctuations are not well understood, and show up in all $< N^2 > - < N >^2$. Avoid them choosing observables such as

- $(\Delta Q)^2$. $\frac{<Q>}{V}$small, so is $\Delta V \frac{<Q>}{V}$ (Jeon, Koch)

- For most other data-points

$$(\Delta N)^2 = V(\Delta \rho)^2 + [\Delta V < N >]^2$$

  So we can measure fluctuations of several quantities $(< (\Delta N_+) >, < (\Delta N_-) >, < (\Delta \pi_+) >, ...)$ and

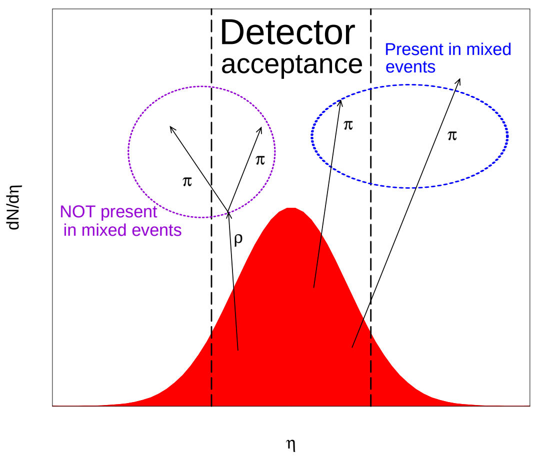
- Fluctuations of ratios(Jeon, Koch), Volume fluctuations irrelevant to 1st order

- fit ΔV (same for all fluctuations)

- understand ΔV (KNO scaling:$(\Delta V)^2 \sim < V >$, pressure ensemble!)

A big problem: Experimental acceptance

subproblem I: Detector response function All measurements depend on rapidity, p_T cuts etc. of detector. For fluctuations, especially of small quantities (such as charge) these effects can dominate

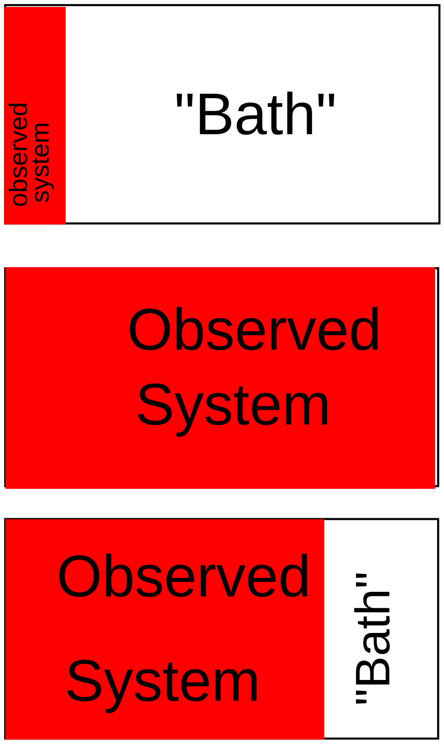


Pruneau, Gavin, Voloshin: use dynamical fluctuations $\sigma_{dyn} = \sigma - \sigma_{stat}$ Where $\sigma_{stat} \sim \frac{1}{<N_1>} + \frac{1}{<N_2>}$ obtained by mixed event technique

σ_{dyn} robust against detector acceptance but needs more parameters ("volume") to be described $\Rightarrow$ no diagrams. Can use it in fit, including one/more yields at same centrality as σ_{dyn} .

But resonances are a problem!

subproblem II: Global conservation laws



GC Ensemble $\omega_N \sim 1$

(+Resonances)

Local,not global Equilibrium

$\langle N\rangle = \langle N\rangle_{GC}$ $\omega_N = 0$

Conservation laws−> Long range correlations

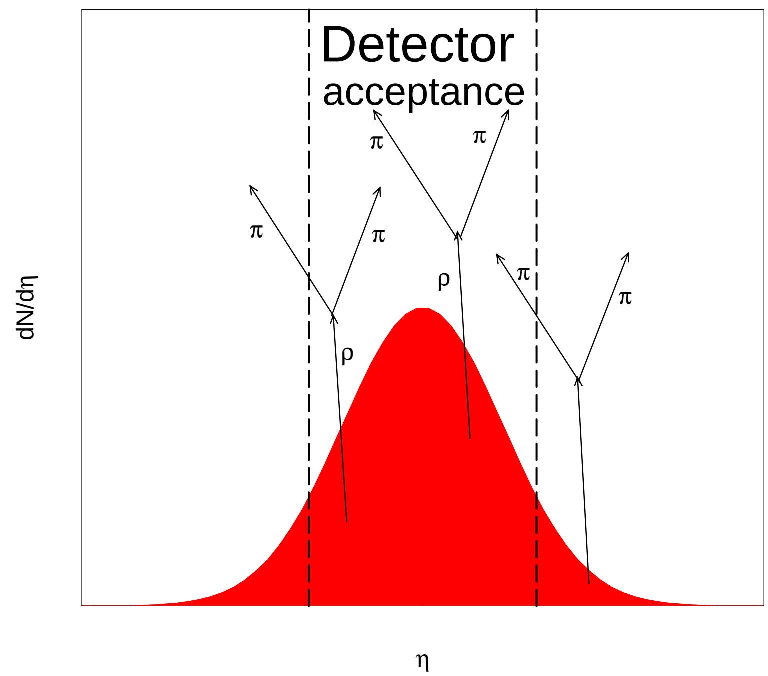
ω_N=??????

Correction coefficient to Grand Canonical esemble (by expanding <u>total</u> enthropy around <u>system</u> conserved number N)

$$\zeta_{GC} = \frac{\langle N\rangle}{2} \frac{(\partial^2 S/\partial N^2)_{N_{\rm tot}}}{(\partial S/\partial N)_{N_{\rm tot}}} \approx \frac{\eta_{exp}}{2\eta_{tot}} \left[\frac{\sum_{n=0}^{\infty} \lambda^n m^2 T K_2\left(\frac{nm}{T}\right)}{\ln\lambda \sum_{n=0}^{\infty} \lambda^n m^2 \frac{T}{n} K_2\left(\frac{nm}{T}\right)} \right]$$

GC description requires $\zeta_{GC} \ll 1$ ($\sim 13\%$ at STAR)

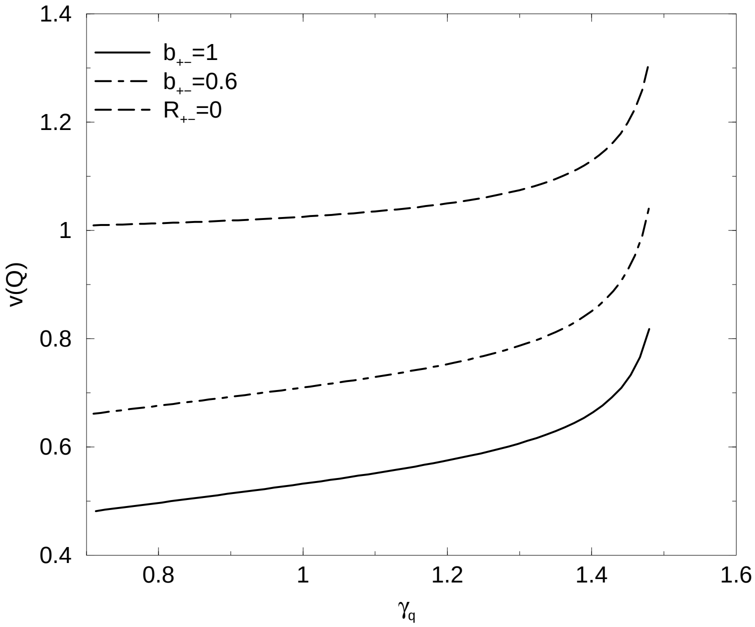
subproblem III: Corrections to correlations due to limited acceptance



$\rho \longrightarrow N^{+}N^{-}$, but detector has limited acceptance. Need fraction of resonances whose decay products are still within acceptance region.
For 2-body decay $\rho \longrightarrow \pi^{+}\pi^{-}$ 3 fractions needed:

- b_{+} N. of positive decay products still in window
- b_{-} N. of negative decay products still in window
- b_{+-} N. of decay products both in window

Same type of arguments in direct reconstruction, except resonance need not be reconstructible



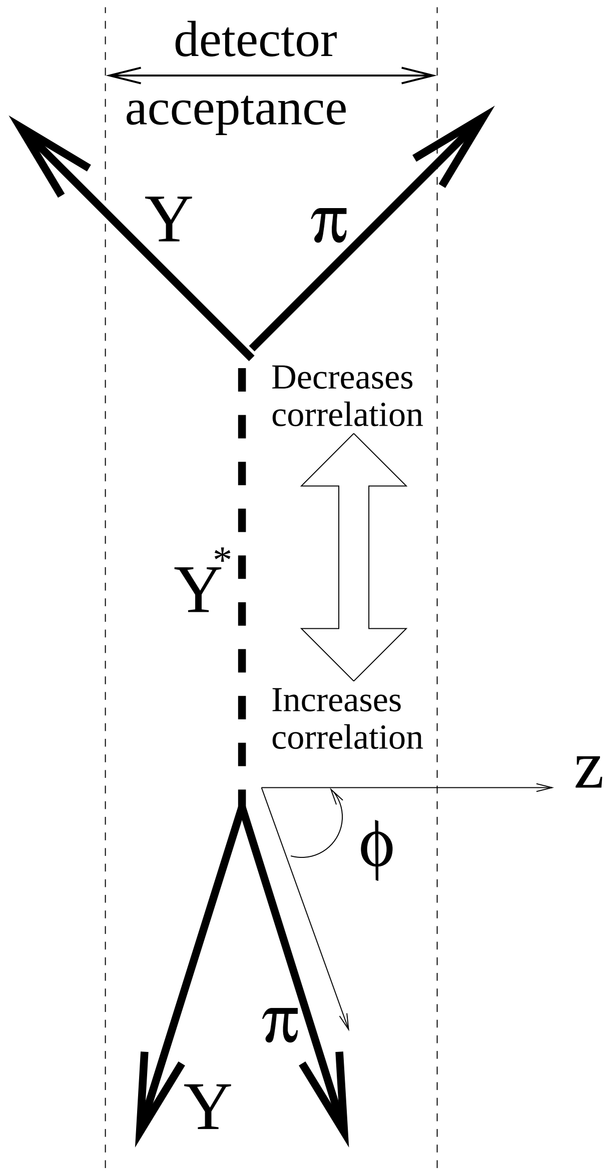
$$\left\langle (\Delta Q)^2 \right\rangle =$$

$$= \left\langle (\Delta N_+)^2 (b_+) \right\rangle + \left\langle (\Delta N_-)^2 (b_-) \right\rangle - 2 b_{+-} \left\langle \Delta N_+ \Delta N_- \right\rangle$$

Boost invariance: $b_+ = b_- = 1$ but $b_{+-} < 1$
since p^* of $\rho \to N_+ N_-$ sets intrinsic rapidity scale!

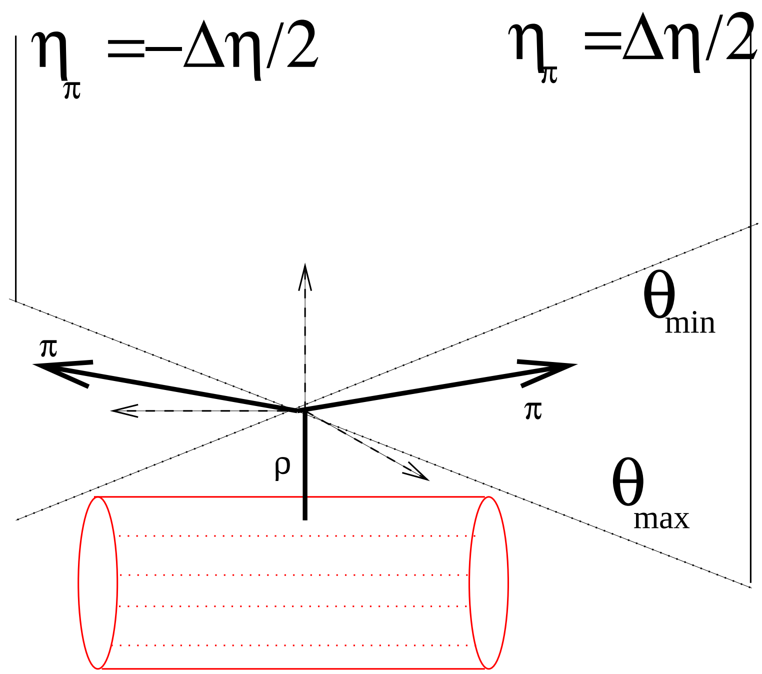
To quantitatively extract T, γ_q, interaction time from fluctuations, b_{+-} has to be calculated for each resonance decay

Good news: Fluctuations still valid T_{chem} probe!



In local-thermal equilibrium Reactions destroying correlation and creating correlation balance out. If physics local, even partial equilibrium should not destroy this balance.

But b_{+-} must still be calculated!



GT, S. Jeon, J. Rafelski, nucl-th/0503026

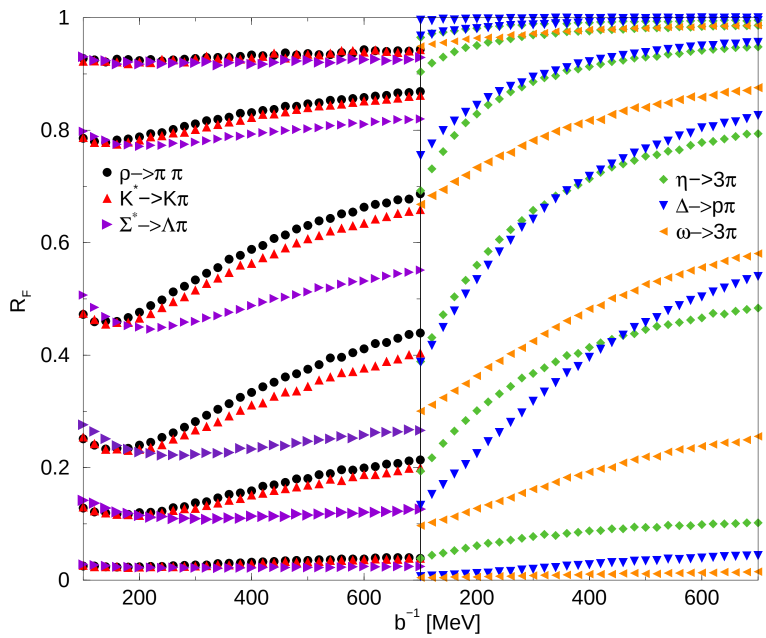
In a thermal-like source the fraction b_{+-} is given by a simple phase space integral

$$b_{+-} = \int_0^\infty dp_{TR} \int_{-\Delta\eta/2}^{\Delta\eta/2} d\eta_R P(\eta_R, p_{TR}) \Omega_{+-}(\eta_R, p_{TR})$$

$$\Omega_{+-}(\eta_R, p_{TR}) = \int \frac{d^3p_+^*}{E_+^*} \frac{d^3p_-^*}{E_-^*} \prod_i \frac{d^3p_i^*}{E_i^*} \Theta_{+-}$$

where:

$$\Theta_{+-} = \Theta_{\eta_+ - \frac{\Delta\eta}{2}} \Theta_{\eta_+ + \frac{\Delta\eta}{2}} \Theta_{\eta_- - \frac{\Delta\eta}{2}} \Theta_{\eta_- + \frac{\Delta\eta}{2}}$$



$$\frac{dN}{dy m_T dm_T} \propto e^{-b^{-1} m_T}$$

- Parameter b includes both temperature and flow
- It needs to be estimated at chemical freeze-out. It's possible since
  - Dependance on b small for most resonance decays
  - Re-interaction tends to increase flow and decrease T, so b not too affected

  Work in progress to put these on quantitative footing

Conclusions: Why fluctuations are good!

Fluctuations, taken together with yields, are a powerful tool of model differentiation. They are capable of:

- Falsifying all statistical models

- Determining experimentally the physically appropriate ensemble in the heavy ion regime

- Together with the direct detection of resonances, directly measure the effect of hadronic reinteractions between chemical and thermal freeze-out.

- Quantitatively determine
  - Freeze-out temperature
  - Non-equilibrium occupation parameters

  And experimentally distinguish between higher temperature equilibriu and super-cooled non-equilibrium freeze-out.

Conclusions: Issues to keep under control before comparing data to (statistical) models

- Experimental acceptance must be small for GC ensemble to be physically appropriate

- Correction coefficients for all leading resonance decays must be estimated

- Volume fluctuations must be kept under control (by choice of observables, fitting, or ansatz such as KNO).

Outlook: What is needed is an open-source Statistical model software capable of describing both yields and fluctuations

SHAREv2.0

http://www.physics.arizona.edu/∼torrieri/SHARE/share.html

Ready and being used for publications. Will be put on the web, hopefully, soon ($\sim weeks$).